If possible suppose a set S have more than one infimum. Let m & m' be two infimum of a set S. so that m & m' are both lower bounds of S. m is g.l.b (infimum) of S and m' is a lower bound of S $\Rightarrow m' \le m$. (i) Similarly m' is the g.l.b (infimum) of S and m is a lower bound of S $\Rightarrow m \le m'$ (ii) by (i) & (ii) $m = m'$.

(infimum) of a set may or may not exist and it may or may not belong to S. It can be easily shown that a set cannot have more than one infimum.

Def^n → The infimum m of a set S has the following two properties:-

(i) m is the lower bound of S. i.e., $m \le x \quad \forall x \in S$.

(ii) No number greater than m can be a lower bound of S. i.e., for any +ve number ϵ, however small, $\exists$ a number $z \in S$ s.t. $z < m + \epsilon$.

$m = \inf S$ (i) m is lower bound of S (ii) for any $\epsilon > 0$ $\exists$ $z \in S$ s.t. $z < m + \epsilon$

Illustrations:-

1. $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$, the set of natural numbers is bounded below but not bounded above, 1 is the lower bound of $\mathbb{N}$. So $\mathbb{N}$ is not bounded.
2. The set I, $\mathbb{Q} = \{ p/q : q \neq 0, p \in I \}$ and $\mathbb{R}$ are not bounded.
3. Every finite set of numbers is bdd.
4. The set S_1 of all positive real numbers $S_1 = \{x : x > 0, x \in \mathbb{R}\}$ is not bounded above, but is the bounded below, the infimum 0 is not a member of the set S_1.